QUANTUM MECHANICS

UNIT-2

VTU University Syllabus (2007-08)

Heisenberg's Uncertainty principle and its physical significance (no derivation). Application of uncertainty principle (Non-existence of electron in the nucleus). Wave function. Properties and physical significance of a wave function. Probability density and normalization of wave function. Setting up of a one dimensional, Time independent Schrödinger wave equation. Eigen values and Eigen functions. Application of Schrödinger wave equation- Energy Eigen values for a free particle. Energy Eigen values of a particle in a potential well of infinite depth.

7-Hours.

Heisenberg's Uncertainty principle

(Derivation from group velocity expression -is not in VTU syllabus)

In classical mechanics, it is possible to determine the position and momentum of a particle simultaneously and exactly, but it is not possible in quantum mechanics. According to a quantum mechanical picture, a moving particle may be considered as a group of waves (wave packet) and the particle may be found anywhere within the wave packet. This indicates that the position of the particle is uncertain within the limits of the wave packet. Moreover, the wave packet has a velocity spread and hence there is uncertainty about the momentum of the particle. For narrow wave packet the position of a particle can be precisely determined but the wavelength and hence the particle's momentum cannot be measured accurately because of lack of enough waves to measure exactly. Conversely for a wide wave packet, the wavelength can be precisely determined but the determination of position of the particle become completely uncertain because of great width of the packet. In 1927 Heisenberg stated the uncertainty involved in position and momentum as below: -

It is impossible to measure precisely and simultaneously both the members of pairs of certain canonically conjugate variables that describe the behavior of an atomic system.

Qauntitatively, the uncertain principle(or the principle of inderminacy) states that the order of magnitude of the product of uncertainties in the simultaneous measurements of the two canonically conjugate variables must be at least of the order of planck's constant /4π i.e., (h/4π).

Mathematically,

$$\Delta p_x \, \Delta x \geq \frac{h}{4\pi} = \frac{\hbar}{2} \qquad or$$

$$\Delta J \, \Delta \phi \geq \frac{h}{4\pi} = \frac{\hbar}{2} \qquad or$$

$$\Delta E_k \, \Delta t \geq \frac{h}{4\pi} = \frac{\hbar}{2}$$

Where Δp_x - Uncertainty in momentum.
Δx - Uncertainty in position } Conjugate pairs

ΔJ - Uncertainty in Angular momentum
$\Delta \varphi$ - Uncertainty in Angular position } Conjugate pairs

ΔE_k - Uncertainty in kinetic Energy
Δt - Uncertainty in time. } Conjugate pairs

$\because \Delta x = v \times \Delta t$ *and* $E = \frac{p^2}{2m}$ *Differentiation gives*

$$\Delta p = \frac{\Delta E}{v}$$

This gives $\Delta E \times \Delta t \geq \frac{\hbar}{2}$

Physical Significance of Uncertainty Principle

Since we cannot know exactly both where a particle is right now and what its momentum is, we cannot say anything definite about where it will be in the future or how fast it will be moving then. We cannot know the future for sure because we cannot know the present for sure. But our ignorance is not total: we can still say that the particle is more likely to be in one place than another and that its momentum is more likely to have a certain value than another. This indeterminacy is inherent in the nature of a moving body. The justification for the many "derivations" of this kind is

- They show it is impossible to imagine a way around the uncertainty principle
- They present a view of the principle that can be appreciated in a more familiar context than that of wave groups.

A particle approach also gives the same uncertainty result.

The uncertainty principle can be arrived at from the point of view of the particle properties of waves as well as from the point of view of the wave properties of particles. We might want to measure the position and momentum of an object at a certain moment. To do so, we must touch it with something that will carry the required information back to us. That is, we must poke it with a stick, shine light on it or be interfered with in some way. If we consider such interferences in detail, we led to the same uncertainty principle.

Suppose we look at an electron using light of wavelength λ. Each photon of this light has the momentum h/λ. When one of these photons bounces off the electron, the electron's original momentum will be changed. The exact amount of the change Δp cannot be predicted, but it will be of the same order of magnitude as the photon momentum h/λ.

$$\Delta p = \frac{h}{\lambda}$$

Hence, longer the wavelength of the observing photon, the smaller the uncertainty in the electron's momentum. Because light is a wave phenomenon as well as a particle phenomenon, we cannot expect to determine the electron's location with perfect accuracy regardless of the instrument used. A reasonable estimate of the minimum uncertainty in the measurement might be one photon wavelength, so that

$$\Delta x \geq h$$

The shorter the wavelength, the smaller the uncertainty in location. If we use light of short wavelength to increase the accuracy of the position measurement, there will be a corresponding decrease in the accuracy of the momentum measurement because the higher photon momentum will disturb the electron's motion to a greater extent. Light of long wavelength will give a more accurate momentum but a less accurate position.

In combining the both we get $\Delta p\,\Delta x \geq h$ this is consistent with $\Delta p\,\Delta x \geq \frac{h}{4\pi}$.

Applications of the Uncertainty Principle

The uncertainty principle based on the law of probability in quantum mechanics explains many facts which could not be explained by classical physics. Some of them are as follows:

1. Non existence of Electron in the Nucleus
2. The radius of the Bohr's First orbit
3. Diffraction of beam of electrons
4. Nuclear Beta decay
5. Binding energy of electron in an atom
6. Nitrogen doping of silicon.
7. Existence of protons and neutrons in the nucleus
8. Minimum energy of a harmonic oscillator. etc.,

Non-existence of Electron in the Nucleus

We know that the radius of the nucleus of any atom is of the order of few fermis, i.e., about 10^{-14}m. If the electron is considered inside the nucleus then the maximum uncertainty in its position will be equal to the diameter of the nucleus, i.e., $2x10^{-14}$m.

$$(\Delta x)_{max} = 2\times10^{-14}\ m$$

According to the uncertainty principle

$$(\Delta p)_{min} \approx \frac{\hbar}{(\Delta x)_{max}\times 2} = 2.634\times10^{-21}\ kg\ m/s$$

The momentum of the electron must be at least comparable to with this uncertainty.

Thus, the minimum kinetic energy of the electron of mass m is given by

$$(E_k)_{min} = \frac{p^2}{2m} = 3.18\times10^{-12}\ J \cong 23.825\ MeV$$

But no electron in an atom is found to possess energy greater than 4MeV. Therefore, the existence of an electron in the nucleus is not possible.

Numericals

1) A microscope using photons is employed to locate an electron in an atom to within a distance of 0.2 A^0. What is the uncertainty in the momentum of electron located in this way? (Ans:- 2.64 x 10^{-24}Kg m/sec)
2) Calculate the smallest possible uncertainty in the position of an electron moving with velocity v= 3 x 10^7m/sec. (Ans:- 0.0193A^0)
3) The speed of an electron is measured to be 5 x 10^3m/sec to an accuracy of 0.003%. Find the uncertainty in determining position of this electron (Ans:- 3.8767 x 10^{-4} m.)
4) An electron is confined to a box of length 1.1 x 10^{-8}m. Calculate the minimum uncertainty in its velocity. (Ans:- 5.26 x10^3m/s)
5) What is the minimum uncertainty in the frequency of a photon whose lifetime is about 10^{-8}sec? ($\Delta\nu \geq 8 \times 10^6\ sec^{-1}$)
6) A certain excited state of hydrogen atom is known to have a life of 2.5 x 10^{-14}sec. What is the minimum error, with which the energy of the excited state can be measured? (Ans: - 0.0131eV.)

Wave function

In water waves, the quantity that varies periodically is the height of the water surface. In sound waves, it is pressure. In light waves, electric and magnetic fields vary. What is that varies in the case of matter waves?

The quantity whose variations make up matter waves is called the wave function, symbol ψ. The value of the wave function associated with a moving body at a particular point x,y,z in space at the time t is related to the likelihood of finding the body there at the time.

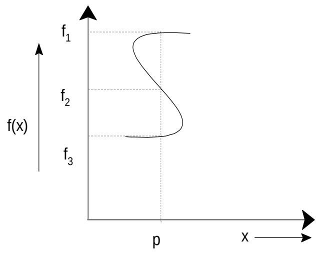
The wave function ψ itself has no direct physical significance. There is a simple reason why ψ cannot by interpreted in terms of an experiment. The probability that something be in a certain place at a given time must lie between 0 (object is definitely not there) and 1(object is definitely there). An intermediate probability, say 0.2, means there is a 20% chance of finding the object. But the amplitude of any wave is negative as often as it is positive and a negative probability, say –0.2 is meaningless. Hence ψ by itself cannot be an observable quantity.

Properties and physical significance of a wave function

To overcome the above discrepancy, a new statistical interpretation of wave function ψ in terms of probability was put forward by Max Born in 1926. According to his view, $|\psi|^2$, the square of the absolute value of the wave function, does not measure the particle density at any point but the probability of finding the particle at that point at any given moment, which is known as probability density and ψ is the probability amplitude.

There is a big difference between the probability of an event and the event itself. Although we can speak of the wave function ψ that describe a particle as being spread out in space, this does not mean that the particle itself is thus spread out. The probability of finding the particle at a given point is proportional to $|\psi|^2$ at that point and the probability of finding the particle within an element of volume dv= dx dy dz is $|\psi|^2$ dv. Since the particle is necessarily somewhere in space, the integral over the whole space must be unity, that is $\int_{-\infty}^{\infty} |\psi|^2 dv = 1$. ***Wave function ψ satisfying the above relation is called normalized wave function. Every* acceptable wave function must be normalisable.** Besides being normalisable, an acceptable state function or wave function ψ must fulfill the following conditions.

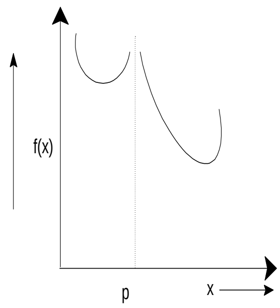
1) **Normalised wave function must be single valued**: If state function ψ has more than one value at any point, it would mean that there are more than one probability of finding the particle at that point which is obviously inaccessible.



Multivalued Function

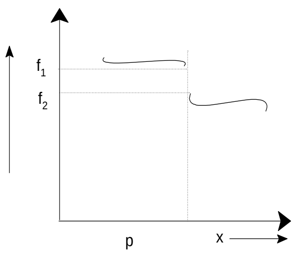
Here if f(x) is wavefunction, then the probabilities of finding the particle has 3 different values at the same location which is meaningless. Hence the wavefunction is not acceptable.

2) **It must be finite everywhere**: Wave function ψ must be finite. If, for instance it is infinite for a particular point, it would mean an infinitely large probability of finding the particle at that point. This would violate Heisenberg's uncertainty principle. Therefore, ψ must be finite or zero everywhere.



Function not finite at a point 'p'

3) **ψ must be continuous throughout the entire space of the system and have a continuous first derivative:** This condition arises from the Schrödinger equation itself, which shows that the second derivative must be finite every where. This can be possible only if first derivative is continuous at any boundary where potential changes. In addition, the existence of first derivative as a continuous function reveals that ψ too is continuous across a boundary.



Discontinuous function

Setting up of a one-dimensional Time independent Schrödinger Wave Equation (or Steady-state Schrödinger Equation)

In 1926, Schrödinger developed an equation to represent matter waves mathematically, which is known as Schrödinger equation. It gives the complete information about the energy, momentum and position of the particle. Consider the simple harmonic wave moving along the x-direction (one-dimensional) is represented as

$$\frac{d^2y}{dx^2}=\frac{1}{v^2}\frac{d^2y}{dt^2} \qquad (1)$$

The solution of this equation is

$$y = Ae^{i(kx-\omega t)} \qquad (2)$$

Where y is the displacement
ω is the angular frequency
A is the Amplitude of the wave
v is the velocity of the wave

The differential wave equation for matter waves in terms of ψ is

$$\frac{d^2\psi}{dx^2}=\frac{1}{u^2}\frac{d^2\psi}{dt^2} \qquad (3)$$

where u is the velocity of matter waves
The solution for this equation is

$$\psi = \psi_0 \; e^{i(kx-\omega t)} \qquad (4)$$

where ψ0 is the amplitude of the matter wave.
Differentiate equation (4) w.r.t 't'; twice

$$\frac{\partial\psi}{\partial t} = -i\omega\psi_0 e^{-i(kx-\omega t)}$$

$$\frac{\partial^2\psi}{\partial t^2} = i^2\omega^2\psi_0 e^{-i(kx-\omega t)} \qquad \because i^2 = -1$$

$$= -\omega^2\psi \qquad (5)$$

Substitute Equation (5) to Equation (3)

$$\frac{d^2\psi}{dx^2} = -\frac{1}{u^2}\omega^2\psi$$

or

$$\frac{d^2\psi}{dx^2} + \frac{1}{u^2}\omega^2\psi = 0 \qquad (6)$$

Here ω=2πν and u=ν.λ
Where ν is the frequency of the matter wave.
Equation (6) becomes

$$\frac{d^2\psi}{dx^2} + \frac{4\pi^2}{\lambda^2}\psi = 0 \qquad (7)$$

Let us consider a particle of mass m moving with a velocity v. The position of the particles can be represented by (x,y,z) in time 't'. For simplification, if we consider the particle is moving only along x-axis then by using de-Broglie wavelength,

$$\lambda = \frac{h}{mv} = \frac{h}{p} \qquad (8)$$

where λ= de-Broglie wavelength

h= Planck's constant
p= momentum of the particle

Substitute the value of λ in Equation (7) then we get

$$\frac{d^2\psi}{dx^2}+\frac{4\pi^2p^2}{h^2}\psi=0 \qquad (9)$$

Let us consider this particle is piloted by the wave function ψ, the total energy in a non-relativistic case is given by

E = Kinetic Energy + Potential Energy

$$E=\frac{1}{2}mv^2+V \qquad (10)$$

If p is momentum of a particle along x-direction then
P=mv, therefore Equation (12) becomes

$$E=\frac{p^2}{2m}+V \qquad (11)$$

or

$$p^2=2m(E-V) \qquad (12)$$

Substitute the value of p2 in Equation (9)

$$\frac{d^2\psi}{dx^2}+\frac{4\pi^2}{h^2}2m(E-V)\psi=0 \qquad (13)$$

or

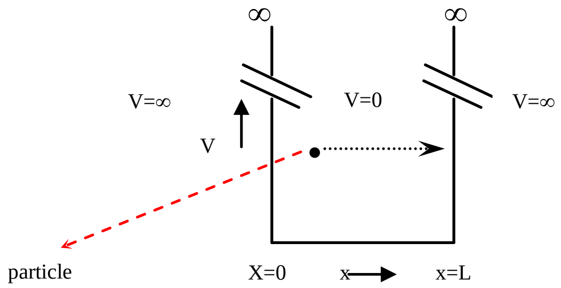
$$\boxed{\frac{d^2\psi}{dx^2}+\frac{2m}{\hbar^2}(E-V)\psi=0}$$

where $\hbar=\frac{h}{2\pi}$ (14)

This is Schrödinger's time independent equation.
The energy values given by Schrödinger wave equation for a particular problem are called ***Eigen values*** and corresponding wave function ψ as ***Eigen functions***.

Applications of Schrödinger wave equation

Energy Eigen values for a free particle. Energy Eigen values of a particle in a potential well of infinite depth.
(or of a particle in a Box):



Particle in an Infinite Potential Well

Mathematical techniques are required to solve the Schrödinger equations. The particle in a potential well is the simplest quantum-mechanical problem. Consider a particle of mass 'm' and charge 'e' is enclosed in a one-dimensional box of height infinity and width L along the x- axis as shown in the above figure. Outside this region, the potential energy V is taken to be infinite, and within this region it is zero. Such a Potential in space is called infinite potential well. A particle bound with in such an infinite potential is also referred to as particle in a box. Assumptions considered are

- Motion of the particle is restricted between x= 0 to x=L
- The particle does not loose energy, when it collides with boundaries.
- Outside the box, the potential energy is infinite.

So, we have two boundary conditions,

1. $v(x) = \infty$ for $x \leq 0$ and $x \geq L$
2. $v(x) = 0$ for $0<x<L$

Let ψ be the wave function associated with the particle.

Outside the well, the Schrödinger time independent equation is

$$\frac{d^2\psi}{dx^2}+\frac{8\pi^2 m}{h^2}(E-\infty)\psi=0 \tag{1}$$

This equation holds well only if $\psi=0$ for all points outside the well, i.e., no particle exists outside the particle.

Inside the well, the Schrödinger time independent equation is

$$\frac{d^2\psi}{dx^2}+\frac{8\pi^2 m}{h^2}(E)\psi=0 \tag{2}$$

i.e.,
$$\frac{d^2\psi}{dx^2}+k^2\psi=0 \tag{3}$$

where
$$k^2 = \frac{8\pi^2 mE}{h^2} \tag{4}$$

Equation (3) is a standard second order partial differential equation. Its solution is either a sine function of a cosine function.

i.e., $$\psi(x)=A\sin(kx)+B\cos(kx) \tag{5}$$

Where A and B are constants. Its value can be obtained by applying the boundary conditions.

i) At x=0, $\psi=0$

Equation (5) becomes $0= A\sin 0+ B\cos 0$

$\therefore B=0$

Equation (5) becomes $$\psi = A\sin(kx) \tag{6}$$

ii) At x=L, $\psi=0$

$\therefore$ Equation (6) becomes $A \sin(kL)=0$

Here, either A=0 or $\sin(kL)=0$

If A=0, the wave function becomes zero. It shows that particle is not located in the box.

Therefore, $\sin(kL) =0$

i.e., $kL= n\pi$

$$\therefore k = \frac{n\pi}{L}$$

where n= 1,2,3,4.........

$$k^2 = \frac{n^2\pi^2}{L^2} \tag{7}$$

Compare the values of k^2 from equation (4) and equation(7)

$$\frac{8\pi^2 mE}{h^2}=\frac{n^2\pi^2}{L^2}$$

or $$E=\frac{n^2\pi^2}{8mL^2} \tag{8}$$

Thus the energy of the particle inside the box is

a. Directly proportional to the square of the natural number i.e., 'n'.
b. Inversely proportional to the square of the length of the box , L^2.
c. Inversely proportional to the mass of the particle.

These values are known the eigen values of n^{th} particle. Since only integral values are possible for 'n', the energy of a particle in one-dimensional potential well is quantized.
With these values of k, Equation (6) changes to

$$\psi = A\sin\left(\frac{n\pi}{L}\right) \tag{9}$$

Normalization of wave function

The integral of wave function over the entire space in the potential well must be equal to unity because, there is only one particle and at any time it is present somewhere inside the well only.

$$\text{i.e., } \int_0^L |\psi_n|^2\, dx = 1 \tag{10}$$

substituting for ψ_n from Equation(9) changes to

$$\int_0^L A^2 \sin^2\left(\frac{n\pi}{L}\right) x\, dx = 1 \tag{11}$$

$$\text{But, } \sin^2(\theta) = \frac{1}{2}(1 - \cos 2\theta) \tag{12}$$

$$\therefore A^2\left[\frac{1}{2}\int_0^L dx - \frac{1}{2}\int_0^L \cos\left(\frac{2n\pi}{L}\right) x\, dx\right] = 1 \tag{13}$$

$$\text{or, } \frac{A^2}{2}\left[x - \frac{L}{2n\pi}\sin\left(\frac{2n\pi x}{L}\right)\right]_0^L = 1 \tag{14}$$

$$\text{or, } \frac{A^2 L}{2} = 1 \qquad (\sin ce \sin(2n\pi\ ) = 0 \tag{15}$$

Thus the normalized wavefunction of a particle in a one-dimensional infinite potential well is given by

$$\psi_n = \sqrt{\frac{2}{L}\sin\left(\frac{n\pi}{L}\right)x} \tag{16}$$

The Hamiltonian operator is

$$\hat{H} = -\frac{h^2}{8\pi^2 m}\frac{d^2}{dx^2} + V \tag{17}$$

Inside the well, i.e., in the region 0 < x < L, the potential V is zero

$$\therefore \quad \hat{H} = -\frac{h^2}{8\pi^2 m}\frac{d^2}{dx^2} \tag{18}$$

The energy eigen value equation is given by

$$\hat{H}\psi = E\psi \tag{19}$$

From equation (8) the lowest acceptable value for n is 1. Consequently the lowest energy corresponds to n acceptable=1 is called the zero-point energy. The zero point energy of an electron in an infinite potential well is given by

$$E_{zero-\text{point}} = \frac{h^2}{8mL^2} \tag{20}$$

The lowest permitted state of energy is referred to as the **ground state energy**. Thus zero point energy is taken as the ground state energy. The states, corresponding to n> 1 are called excited states.

Wave functions, Probability densities and Energy levels for particle in an infinite potential well:

From Equation (16), we can write the eigen functions ψ_1,ψ_2,ψ_3,.........for particle in the well by putting n=1,2,3,......... respectively in the equation.

Case (i), n=1:-

This is the ground state and the particle is normally found in this state. For n=1, the eigen function is $\psi_1 = A\sin\left(\frac{\pi}{L}\right)x$, form Equation (16). In the above equation, ψ_1 for both x=0 and x=L. But ψ_1 has maximum value for x=(L/2). Thus a plot of ψ_1 versus x will be as shown below in figure (a). A plot of $|\psi_1|^2$, the probability density versus x is shown in the above figure (b). It indicates the probability of finding the particle at different locations inside the well. It can be seen in the figure that $|\psi_1|^2$ is 0 both at x=0 and at x=L. It is maximum at x=L/2. It means to say that, in the ground state the particle cannot be found at the walls of the well, and the probability of finding it is maximum at the central region. The energy of the ground state is given by equation (8) at n=1.

$$E_1 = \frac{h^2}{8mL^2}$$

Case (ii), n=2:-

This is the first excited state. The eigen function for this state is

$$\psi_2 = A\sin\left(\frac{2\pi}{L}\right)x$$

Now, ψ_2=0 for the values x=0, L/2, L. Also ψ_2 reaches maximum for x= L/4 and 3L/4. These facts are seen in the plot of ψ_2 versus x shown below in figure (a). The plot of $|\psi_2|^2$ versus x is shown in above figure (b) shows that $|\psi_2|^2$ =0 at x=0, L/2, L. It means that in the first excited state the particle cannot be observed either at the walls, or at the center.

Further for n=2 the energy in the first excited state is 4 time the zero-point energy.

Case (iii), n=3:-

We have the eigen function for the second excited state as $\psi_3 = A\sin\left(\frac{3\pi}{L}\right)x$

ψ_3 **=0, for x=0, L/3, 2L/3, and L.** ψ_3 will have maximum value for x=L/6, L/2 and 5L/6. These factes are revealed in the following figure (a). The plot of $|\psi_3|^2$ versus x shown above has maxima at x=L/6, L/2 and 5L/6 which also imply the locations at which the particle is most likely to be found. This case corresponds to the second excited state. As in the earlier cases the energy in this state is given by $E_3 = 9E_0$.

For a particle with constant energy E but confined in the well, n depends solely on 'L'. Hence, as $L \rightarrow \infty$, $n \rightarrow \infty$. Which essentially means that a free particle can have any energy i.e., the energy eigen values or the possible values of energy are infinite in number. All the permitted energy values are continuous. All these mean, there is no discreteness in the allowed energy values. In other words, there is no quantization of energy in the case of a free particle and the problem is dealt in classical mechanics. Thus a free particle is a "classical entity".

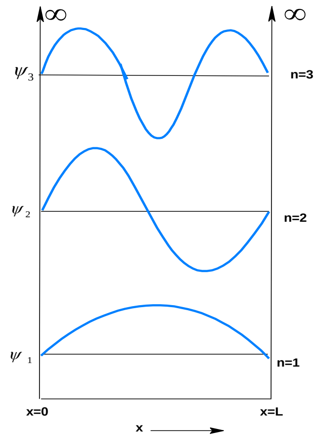


Figure (a) :

Plot of Wave function versus x

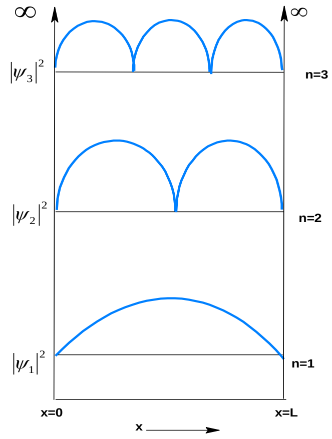


Figure (b) :

Plot of probability density versus x

Eigen Values and Eigen Functions:-

Eigen functions are those wave functions of quantum mechanics which possess the properties that they are single valued and finite everywhere, and also their first derivatives with respect to their variables are continuous everywhere.
Quantum mechanics gives rules for extracting information from ψ to predict the results of measurements. The rule consists for certain mathematical operations, which are to be performed on ψ in designed ways. The specific type of operation need to be performed is identified through what is known as operators.

Table of operators (Not in the VTU Syllabus)

Physical Observable	Operator	Operator Symbol
Momentum	$\frac{h}{2\pi i}\frac{\partial}{\partial x}$	$\hat{p}$
Kinetic Energy	$-\frac{h^2}{8\pi^2 m}\frac{\partial^2}{\partial x^2}$	$\hat{T}$
Total Energy (Also called Hamiltonian)	$-\frac{h^2}{8\pi^2 m}\frac{\partial^2}{\partial x^2}$ + U	$\hat{H}$
Position	x	$\hat{x}$

Quantum mechanical operators could be used to evaluate the physical observable such as energy in which case each of the eigen functions provide one energy value. Since there is only a restricted set of eigen functions, there is also restricted set of energy values. $\hat{A}\psi = \lambda\psi$ Where $\hat{A}$ is the operator for the physical quantity, and ψ is eigen function. The values obtained for a physical observable are called eigen values. If it is energy operator operating on ψ, then they are called energy eigen values. If it is momentum operator that operates on ψ, then they are called momentum eigen values…so on.

Numericals:-

1. Find the energy of an electron moving in one dimension in an infinitely high potential box of width 1.0 A^0. (Ans:- 37.694 n^2 eV)
2. An electron is bound in a one dimensional potential box which has a width 2.5 x 10^{-10}m. Assuming the height of the box to be infinite, calculate the two lowest permitted energy values of the electron. (ans:- 6.04eV and 24.16eV)
3. Calculate the lowest energy of a neutron confined to the nucleus, where nucleus is considered a box with a size of 10^{-14}m. (Ans:- 6.15Mev)

4. A particle is in motion along a line between x=0 and x=L with zero potential energy. At points for which x<0 and x>L, the potential energy is infinite. The wave function for the particle in the n^{th} state is given by

$$\psi_n = A\sin\left(\frac{n\pi x}{L}\right)$$

Find the expression for the normalized wave function.

5. Show that $\phi(x) = e^{ikx}$ is acceptable eigen-function, where k is some finite constant. For a region $-a \le x \le a$; normalize the given eigen-function.
6. A particle is moving in one dimensional potential well of infinite height and of width 25A⁰. Calculate the probability of finding the particle in an interval of 5 A⁰ at a distances of a/2, a/3 and a , where a is the width of the well assuming that the particle is in its least state of energy.(Ans:- P_1=0.3871, P_2= 0.2937 and P_3=0.0486)

Questions:-

1. State and explain the Heisenberg uncertainty principle. Using this principle, show that the electrons cannot reside in an atomic nucleus.
2. State the exact statement of Heisenberg uncertainty principle. Name three pairs of physical variables for which this law holds true.
3. Derive time-independent Schrödinger wave equation. What is the physical significance of state function 'ψ' used in this equation? What conditions must it fulfil?
4. Write down the Schrödinger equation for a particle in one-dimensional box. Obtain the eigen functions and eigen values for this particle.
5. A particle is moving freely within one-dimensional potential box. Find out the eigen functions of the particle and show that it has discrete eigen values.
6. Find the expression for the energy state of a particle in one-dimensional box.
7. What do you mean by an operator? Write the operators associated with energy and momentum.
8. Give an analytical discussion of Gamma Ray microscope experiment to prove the inevitability of uncertainty principle.
9. Set up time-independent one-dimensional Schrödinger equation.
10. What is normalization of a wave function? What are the physical significance and properties of wave function?
11. What are eigen values and eigen functions? Discuss the nature of eigen values and eigen functions.
12. Give the Max Born's interpretation of wave function and explain the normalization conditions.
13. Solve the Schrödinger wave equation for the allowed energy values in the case of particle in a box.
14. Using Schrödinger wave equation for a particle in one-dimensional well of infinite height, discuss about energy eigen values.
15. Describe zero-point energy.
16. Assuming the time independent Schrödinger wave equation, discuss the solution for a particle in one-dimensional potential well of infinite height. Hence obtain the normalized wave function.
17. What is the physical interpretation of wave function? How a free particle wave function signifies a particle in space and momentum?
18. Solve the Schrödinger wave equation for the one-dimensional potential well defined by

$$V(x) = \infty \qquad x < 0 \quad and \quad x > L$$
$$V(x) = 0 \qquad 0 \le x \le L$$